Las colecciones de Documentos de Trabajo del CIDE representan un medio para difundir los avances de la labor de investigación, y para permitir que los autores reciban comentarios antes de su publicación definitiva. Se agradecerá que los comentarios se hagan llegar directamente al (los) autor(es). D.R. 2002, Centro de Investigación y Docencia Económicas, A. C., carretera México Toluca 3655 (km.16.5) ,Lomas de Santa Fe, 01210 México, D. F., tel. 727-9800, fax: 292-1304 y 570-4277. Producción a cargo del (los) autor(es), por lo que tanto el contenido como el estilo y la redacción son responsabilidad exclusiva suya.
10 de diciembre de 2002

NÚMERO 247

Arturo Antón

TAX REFORM IN AN ENDOGENOUS GROWTH MODEL WITH MONEY
Abstract

We study the welfare effects of alternative feasible tax reforms in a Uzawa-Lucas model with money, cash-credit goods and distorting taxes on consumption, physical capital and labor income. The representative household must use money in order to purchase cash goods and pay taxes, where taxes are needed to finance an exogenous stream of government spending. A tax reform is designed so that a tax on capital income is gradually decreased to zero and replaced either by higher taxes on labor income or money holdings. If the analysis is restricted to the balanced-growth path, numerically we find sizable welfare gains of such a tax reform regardless of how lost revenue is raised. Once the initial distortion on consumption and worked hours is taken into account, the welfare cost of the tax reform may increase when lost revenue is raised through seigniorage. It turns out that the share of cash goods in total consumption is crucial for explaining whether welfare is increasing or decreasing along the transition.

Resumen

En este artículo se consideran los efectos en bienestar de una reforma fiscal utilizando el modelo de Uzawa-Lucas con dinero, bienes de consumo a crédito y en efectivo, así como impuestos al consumo, capital físico y trabajo. El agente representativo utiliza el dinero para comprar bienes de consumo y para pagar impuestos, donde los impuestos son necesarios para financiar una secuencia exógena de gastos de gobierno. La reforma fiscal consiste en disminuir gradualmente el impuesto al capital a cero para ser reemplazado ya sea con mayores impuestos al trabajo o a la tenencia de saldos reales. Si el análisis se restringe al sendero de crecimiento balanceado, numéricamente se encuentran grandes ganancias en bienestar a consecuencia de la reforma fiscal, sin importar cómo se financia el gobierno. Sin embargo, una vez que se toman en cuenta las distorsiones iniciales sobre el consumo y las horas destinadas al trabajo, el costo en bienestar de la reforma fiscal puede incrementarse cuando se utiliza mayor señoreaje. Se muestra cómo la proporción de los bienes de consumo en efectivo sobre el consumo total es crucial para determinar si el bienestar aumenta o disminuye a lo largo de la transición.
Over the past years there has been a considerable effort to measure the welfare cost of different types of taxation in a reliable way, including taxes on money holdings. In general, it is possible to identify two different (but somehow related) branches in a dynamic general equilibrium, representative-agent context. On the one hand, there exist models that quantify the welfare cost of capital income taxation, labor taxation, or both, but abstracting from the use of money. Examples of this type of models include Chamley (1981), King and Rebelo (1990), Lucas (1990), Jones et al. (1993) and Ortigueira (1998). On the other hand, alternative frameworks have been proposed to seize the welfare cost of inflation under a first-best, perfect-foresight general equilibrium economy. Examples of such efforts are found in Cooley and Hansen (1989), Jones and Manuelli (1995), and Dotsey and Ireland (1996), among others. However, the welfare effects of monetary and fiscal policies altogether have been studied only in a few papers, notably Cooley and Hansen (1991, 1992) and Wen and Love (1998). In particular, Cooley and Hansen (1992) find that the best policy in welfare terms is not to tax capital at the steady state but using instead other types of distorting taxes in a standard exogenous growth model.

This paper belongs to the last class of papers. Specifically, we extend the analysis of Cooley and Hansen (1992) to allow for endogenous growth in a neoclassical model. By doing so we are basically interested in two issues: First, we want to know whether replacing a tax on physical capital for other tax instruments is still welfare improving. Second, given such a tax policy, we inquire which among the alternative tax instruments available brings the lowest distortion in welfare terms. The reason for choosing such type of policies is strongly motivated by the results from the optimal tax literature that capital should not be taxed at all in the long run (e.g. Chamley (1986), Lucas (1990), Chari et al. (1991), Jones et al. (1993)).

The framework of our model is the familiar two-sector endogenous growth model based on Uzawa (1965) and Lucas (1988) with preferences of the type postulated by Lucas and Stokey (1987), where a representative household with perfect foresight derives utility from the consumption of cash and credit goods. Money enters the model via an augmented cash-in-advance constraint (to be described later). In addition, there is a government that needs to impose taxes on

---

1 I would like to thank Salvador Ortigueira for his continuous support and comments on earlier versions of this paper. I also kindly acknowledge financial support from CONACYT. Usual disclaimer applies.

1 The advantage of using this model is that the endogenous growth rate along the balanced growth path is not affected by taxes of any kind (cf. Ortigueira (1998)). Therefore, we believe this makes our analysis more transparent in order to avoid the huge effects of taxation on both growth rates and welfare found elsewhere (cf. King and Rebelo (1990), Jones et al. (1993)).
labor and (physical) capital income, consumption and money holdings to finance her spending needs.

Our paper differs from Cooley and Hansen (1992) in three important ways. First, the adoption of the Uzawa-Lucas endogenous growth model with money allows inflation to affect the allocation of time between productive and schooling activities along the transition path and thus distort the accumulation of human capital. Second, we incorporate into our model the general idea derived from the overlapping generations literature that taxes must be paid with money (cf. Balasko and Shell (1981)). Under this assumption, money may affect the physical-human capital ratio along the balanced-growth path without affecting the endogenous growth rate. This extra assumption creates an additional distortion from seigniorage policies that is missing in more standard cash-in-advance models. Finally, we allow for more general preferences so that cash and credit goods are non-separable.

We perform tax reform exercises where lost revenue derived from a lower tax on physical capital income is replaced by other sources of revenue like money creation or labor income taxes. We restrict such tax reforms so that the present value of government revenue equals the present value of a stream of government spending. Our results show that, given a decrease in the tax on physical capital income, there are unambiguously sizable welfare gains along the balanced growth path regardless of how lost revenue is raised. In contrast, we find that eliminating a capital income tax and replacing lost revenue with a tax on money holdings is not necessarily welfare improving once transitional dynamics effects are taken into account. Although our extra assumptions—the endogenous growth context and the need to pay taxes with money—bring substantial differences in welfare estimates with respect to the standard exogenous growth case, the welfare increasing result does not hinge on them. It turns out that the share of cash goods in total consumption plays a crucial role in explaining whether welfare is increasing or decreasing along the transition.

Contrary to what is found in Cooley and Hansen (1992), in our model the tax reform mentioned above yields a two percent point increase in the welfare cost of taxation under the benchmark case when measured in consumption terms. This welfare cost is far from being disregarded: for the U.S. economy, a 2 percent decrease in consumption would translate into a loss of around $125 billion.

The paper is organized as follows. Section 2 describes the model. Properties along the balanced-growth path as well as along the transition path are discussed. Section 3 includes the calibration of the model and the welfare costs of alternative tax policies along both the balanced growth path and the transition path. Section 4 concludes.
2 The model

2.1 The Economic Environment

We consider an infinite horizon, deterministic economy consisting of a representative household with perfect foresight. The household derives utility from the consumption of two types of goods: a “cash” good \( c_1(t) \) and a “credit good” \( c_2(t) \), as in Lucas and Stokey (1987). For simplicity, the utility function takes the particular form:

\[
U[C(t)] = \int_0^\infty e^{-\rho t} \frac{(C(t))^{1-\sigma} - 1}{1-\sigma} dt, \quad \sigma \neq 1
\]

and \( U[C(t)] = \log C(t) \) if \( \sigma = 1 \), where \( \rho > 0 \) is the discount rate, \( \sigma > 0 \) is the inverse of the intertemporal elasticity of substitution and \( C(t) \) denotes aggregate consumption, which is a function of cash and credit goods consumed by the household. To allow for an aggregate consumption function as general as possible, we follow Chari et al. (1991), Wright (1991) and Jones and Manuelli (1995) and consider the following family of constant elasticity of substitution consumption functions:

\[
C(c_1(t), c_2(t)) = (\eta c_1(t)^\psi + (1-\eta)c_2(t)^\psi)^{1/\psi}
\]

where \( 1/(1-\psi) \) is the elasticity of substitution between cash and credit goods with \( \psi < 1 \). Accordingly, cash and credit goods are complements (substitutes) if \( \psi < 0 \) (\( 0 < \psi < 1 \)). Note that (2) simplifies to \( C(c_1(t), c_2(t)) = c_1(t)^\eta c_2(t)^{\psi-\eta} \) when \( \psi = 0 \). Thus the type of utility function in (1) - (2) conveniently nests the functional form used in Cooley and Hansen (1992) for \( \psi = 0 \) and \( \sigma = 1 \) when labor is inelastically supplied. As mentioned later, consumption of the credit good may loosely be reinterpreted as leisure, as suggested by Lucas and Stokey (1987).

There are three types of assets in the model: physical capital \( k(t) \), human capital \( h(t) \) and real money balances \( m(t) \). The household owns the stock of physical capital \( k(t) \) and is endowed with a (normalized) unit of time that may be devoted either to work or to “schooling” activities. For simplicity, we assume that the fiscal authority imposes flat rate taxes (in terms of the consumption good) on the consumption of cash goods, physical capital and labor income, denoted as \( \tau_c, \tau_k \) and
Since total resources are either consumed or invested, the budget constraint for the household is given by:

\[(1 + \tau_c) c_1(t) + c_2(t) + i(t) + x(t) = (1 - \tau_k) r(t) k(t) + (1 - \tau_i) w(t) u(t) h(t) + \tau_k \delta k(t) + T(t) \quad (3)\]

where \(r(t)\) is the gross rate of return on physical capital, \(w(t)\) is the gross wage rate per effective unit of human capital, \(u(t)\) denotes the supply of working time, \(\tau_k \delta k(t)\) is the depreciation allowance built into the tax code and \(T(t)\) are lump-sum transfers from the government.\(^3\) The term \(x(t) = m(t) + \pi(t) m(t)\) in (3) represents gross investment in real money balances (where \(\pi(t)\) denotes the inflation rate) and \(i(t)\) is gross investment in physical capital defined in the standard way:

\[i(t) = \dot{k}(t) + \delta k(t) \quad (4)\]

When not at work, the representative agent may increase her stock of human capital \(h(t)\) by devoting \(1 - u(t)\) units of time to activities outside the market (e.g., schooling activities). For simplicity, we allow physical and human capital to depreciate at the constant rate \(\delta\). If we let \(B\) denote the marginal productivity of schooling time, the law of motion for human capital is then given by:

\[h(t) = B(1 - u(t)) h(t) - \delta h(t) \quad (5)\]

Money is valued in this economy because it is required to purchase units of the cash good \(c_1(t)\) and pay taxes net of government expenditures and transfers. This latter assumption roughly incorporates the idea from the overlapping-generations literature that taxes must be paid with money (Balasko and Shell (1981)). In our framework, it implies that the household needs to pay taxes before receiving income from labor and capital services. In other words, the representative consumer needs to carry extra money holdings well in advance in order to cover all her net tax obligations. Thus the modified cash-in-advance constraint may be expressed as:

\[(1 + \tau_c) c_1(t) + \tau_k (r(t) - \delta) k(t) + \tau_i w(t) u(t) h(t) \leq m(t) + G(i) + T(t) \quad (6)\]

\(^2\) We assume that the credit good is not taxed for two reasons: First, as discussed later, it greatly simplifies the cash-in-advance constraint at equilibrium; and, most important, this assumption naturally arises if we alternatively interpret the credit good as a non-market activity such as leisure.

\(^3\) We follow Cooley and Hansen (1992) for the inclusion of depreciation allowances measured in real terms. Although nominal depreciation allowances would be a more realistic assumption, this would make the model more cumbersome. Jones and Manuelli (1995) discuss the effects of including nominal depreciation allowances in an endogenous growth model similar to ours.
Production of the single homogenous good $y(t)$ is given by a standard Cobb-Douglas production function of the form $y(t) = Ak(t)^{\alpha}(u(t)h(t))^{1-\alpha}$, where parameters $A$ and $\alpha$ satisfy $A > 0$ and $0 < \alpha < 1$. Profit maximization entails perfectly competitive firms to pay each factor of production according to their marginal productivities, namely:

$$r(t) = \alpha Ak(t)^{\alpha-1}(u(t)h(t))^{1-\alpha},$$  

$$w(t) = (1-\alpha)Ak(t)^{\alpha}(u(t)h(t))^{-\alpha}.$$  

The solely role of government in this economy is to provide currency and to impose taxes on (physical) capital income, labor income, money holdings and consumption of the cash good in order to finance a stream of lump-sum transfers $T(t)$ and exogenous government expenditures $G(t)$. We allow both $T(t)$ and $G(t)$ to grow at the same rate as the economy does so that these variables do not become a negligible fraction of output over time. As in Cooley and Hansen (1992), we assume that the government is fully and credible committed to pursue the tax policy announced.

Money is issued by the government at the constant rate $\mu$, i.e., $\mu = M(t)/M(t)$, where $M(t)$ is the (nominal) money supply. Equilibrium in the money market is thus reached when the nominal price level $P(t)$ is such that real money demand equals real money supply, $m(t) = M(t)/P(t)$. Therefore, it must hold that:

$$m(t)/m(t) = \mu - \pi(t)$$  

The amount of revenue raised by the government through money creation at time $t$ is just $M(t)/P(t) = \mu m(t)$. Therefore, at each point in time the government’s budget constraint must satisfy:

$$G(t) + T(t) = \mu m(t) + \tau_k (r(t) - \delta)k(t) + \tau_i w(t)u(t)h(t) + \tau_e c_1(t)$$  

### 2.2 Competitive Equilibrium

Given $k(0) = k_0$ and $h(0) = h_0$, a competitive equilibrium is defined as the set of infinite sequences for quantities $\{c_1(t), c_2(t), k(t), h(t), m(t), u(t), i(t), x(t), G(t), T(t)\}$, factor prices $\{r(t), w(t)\}$ and constant tax policy $\{\tau_k, \tau_i, \tau_e, \mu\}$, such that:
(i) Taking factor prices and taxes as given, the sequence \(\{c(t), c_2(t), k(t), h(t), m(t), u(t), l(t), x(t)\}\) maximizes (1) subject to (2) – (6);

(ii) The sequence \(\{k(t), h(t), m(t), u(t), r(t), w(t), G(t), T(t)\}\) satisfies equations (7), (8) and (10); and

(iii) The market clearing condition for the goods market

\[c_1(t) + c_2(t) + i(t) + G(t) = Ak(t)^2 (u(t)h(t))^{\alpha - 1}\]

holds.

The definition of a competitive equilibrium implies that the following first-order conditions (plus some well-known transversality conditions) for the program considered above need to be satisfied:

\[
\begin{align*}
U_{c1}(t) &= (1 + \tau_c)(\lambda_1(t) + \lambda_2(t)) \quad (11a) \\
U_{c2}(t) &= \lambda_3(t) \quad (11b) \\
\lambda_1(t) &= \lambda_3(t) \quad (11c) \\
(1 - \tau_l)\lambda_1(t)w(t) - \tau_l\lambda_2(t)w(t) &= B\lambda_4(t) \quad (11d) \\
\lambda_2(t)[(1 + \mu)m(t) - c_1(t)] &= 0, \quad \lambda_2(t) \geq 0 \quad (11e) \\
\dot{\lambda}_1(t) &= [\rho + \delta - r(t) + \tau_k(r(t) - \delta)(1 + \lambda_2(t)/\lambda_1(t))]\lambda_1(t) \quad (11f) \\
\dot{\lambda}_3(t) &= [\rho + \pi(t) - \lambda_2(t)/\lambda_3(t)]\lambda_3(t) \quad (11g) \\
\dot{\lambda}_4(t) &= (\rho + \delta - B)\lambda_4(t) \quad (11h)
\end{align*}
\]

where \(U_i(t)\) denotes the derivative with respect to the \(i\)-th argument, \(\lambda_1(t), \lambda_3(t)\) and \(\lambda_4(t)\) are the corresponding co-state variables for physical capital, money and human capital, and \(\lambda_2(t)\) is the Lagrange multiplier for the extended cash-in-advance constraint. Most of the first order conditions have the standard interpretation so we do not pursue further on them. From (11d) it is readily noticed how the assumption that taxes must be paid with money introduces an extra cost of working since the

\[1\] The standard transversality condition \(B > \delta + \rho\) is assumed throughout this paper, where \(\rho\) is the endogenous growth rate (to be defined later).
Arturo Antón/Tax Reform in an Endogenous Growth Model with Money

household needs to increase her money holdings in response. By a similar reason, the otherwise well-known rule of optimal saving with taxes (equation (11f)) now includes the cost of holding money.

As is well known, the cash-in-advance constraint holds with equality whenever the nominal interest rate is positive. This condition is satisfied if the growth rate of money is sufficiently large at each point in time, namely if μ is such that:

\[ \mu > \frac{\dot{m}(t)}{m(t)} - (r(t) - \delta)(1 - \tau_k) \]

Along the paper we assume this is always the case. Accordingly, since (11e) implies \( \dot{m}(t)/m(t) = \dot{c}_1(t)/c_1(t) \) for constant μ, the inflation rate is endogenously determined by:

\[ \pi(t) = \mu - \dot{c}_1(t)/c_1(t) \] (12)

Finally, by manipulating first-order conditions it may be shown that the ratio of credit to cash goods is given by:

\[ \frac{c_2(t)}{c_1(t)} = \frac{\eta [1 + \tau_k (r(t) - \delta)]}{(1 - \eta)(1 + \tau_c)(1 + \pi(t) + r(t) - \delta)} \] (13)

As equation (13) illustrates, either a higher consumption tax or a higher inflation rate tilts consumption toward credit goods. On the other hand, a higher tax on capital income decreases the credit-cash good ratio since now higher money holdings are required to face the new tax obligation. Given that real money holdings and consumption of cash goods are proportional in equilibrium, the credit-cash good ratio should decrease accordingly.

2.3 Balanced Growth Path

Along a balanced-growth path, quantities \( c_1(t), c_2(t), i(t), x(t), k(t), h(t), m(t) \) grow at constant rates and \( u(t) \) remains constant. Therefore,

\[ \frac{\dot{c}_1(t)}{c_1(t)} = \frac{\dot{c}_2(t)}{c_2(t)} = \frac{\dot{k}(t)}{k(t)} = \frac{\dot{h}(t)}{h(t)} = \frac{\dot{m}(t)}{m(t)} = g \]

^5 To see this, manipulate (9), (11c), (11e) and (11f) to find that \( \lambda_4(t) > 0 \) if and only if the above condition holds.
and \( u(t)/u(t) = 0 \). From first-order conditions, a balanced-growth path is characterized by the following system of equations:

\[
\begin{align*}
\rho + \sigma g &= (r - \delta) \left[ \frac{1 - \tau_k (1 + \mu - g)}{1 + \tau_k (r - \delta)} \right], \\
(r - \delta) \left[ \frac{1 - \tau_k (1 + \mu - g)}{1 + \tau_k (r - \delta)} \right] &= B - \delta, \\
\frac{c_1}{k} + \frac{c_2}{k} + \frac{G}{k} + g &= A k^{\alpha - 1} (uh)^{1-\sigma} - \delta, \\
g &= B(1 - u) - \delta, \\
\frac{c_2}{c_1} &= \Omega - 1.
\end{align*}
\]

where we have used the fact that \( \pi = \mu - g \) and \( \Omega \) is given by

\[
\Omega = 1 + \left( \frac{\eta}{(1 - \eta)(1 + \tau_k)(1 + \mu + B - \delta - g)} \right)^{\nu/(\nu - 1)}
\]

As is well-known, system (14a)-(14e) may be solved in terms of control-like variables \( c_1/h \) and \( c_2/h \), the state-like variable \( k/h \), worked hours \( u \) and the growth rate \( g \), taking \( G/h \) as given. The system is thus conveniently solved in a recursive way. From (14a) and (14b), it may be readily shown that the endogenous growth rate of the economy \( g \) is given by:

\[
g = \frac{B - \rho - \delta}{\sigma}
\]

This is just the standard result for a non-monetary version of the Uzawa-Lucas model with taxes (cf. Ortigueira (1998)). Once the value for \( g \) is known, a unique value for \( u \) is given by (14d). Then, the physical-human capital ratio \( (k/h)^* \) is obtained from (14a). Finally, manipulation of (14c) and (14e) yields \( (c_1/h)^* \) and \( (c_2/h)^* \).

At this point, it is important to remark that neither taxes nor the growth rate of money affect both the growth rate and worked hours. However, it may be shown
that an increase in either $\tau_k$ or $\mu$ unambiguously decreases $(k/h)^*$. The assumption that money is needed to pay taxes on capital income is crucial for the latter result: a higher rate of money growth increases the shadow price of physical capital while keeping unaltered the shadow price of human capital. In order to restore equilibrium along the balanced growth path, the ratio of physical to human capital must be decreased accordingly. If $\tau_k = 0$.

On the other hand, we find that labor income taxes do not affect the physical-human capital ratio or any other variable along the balanced-growth path. This is just a well-known property of the Uzawa-Lucas model with taxes and inelastic labor (cf. Ortigueira (1998)). As expected, the tax on capital income and the money growth rate also impact negatively the ratio $(c_l/h)^*$. Finally, in general it is possible to show that the growth rate of money has an ambiguous effect on the credit good-human capital ratio $(c_2/h)^*$. This latter result is partially explained from the non-separability assumption between cash and credit goods given by (2), as discussed in Benabou (1991).

2.4 Transitional Dynamics

To describe transitional dynamics, we redefine variables so that $z(t) = k(t)/h(t)$, $x_1(t) = c_1(t)/h(t)$ and $x_2(t) = c_2(t)/h(t)$. Hence, transition in this economy may be reduced to a 4×4 dynamic system in $z$, $u$, $x_1$ and $x_2$. Contrary to similar models with no money (cf. Ortigueira (1998)), it may be shown that a labor income tax is able to affect the transition path in this economy. This result arises from the assumption that taxes must be paid in advance with money (see equation (11d)). Similarly, changes in the growth rate of money may affect the transition path through its effects on the growth rate of cash goods as well as on the law of motion for the shadow price of physical capital.

The next step is to check whether the system is (locally) saddle-path stable with a unique stable manifold. Due to the relative complexity of the system, an analytical proof is not readily available. However, since our model has nice utility and production functions (i.e., the model exhibits a constant elasticity of substitution utility function and neither externalities nor increasing returns to scale are present), we may infer that in fact that is the case. Actually, for the parameter space considered in this paper we always find a unique negative eigenvalue for all the numeric exercises involved.

* If money is not needed to pay for income taxes, it is possible to show that equation (14b) simply reduces to $(1 - \tau_k)(\rho - \delta) = B - \delta$ so that $(k/h)^*$ is independent of the growth rate of money $\mu$. 
To gain some additional insight into the dynamics of the system, it is also important to figure out the direction of convergence to the balanced growth path. As shown in Caballe and Santos (1993) for a non-monetary, non-taxed version of this model, there are three possible growth cases:

**a) The normal case:** If the economy starts with a higher (lower) physical-human capital ratio than the stationary solution, then the economy moves toward a balanced growth path with a higher (lower) level of human capital.

**b) The paradoxical case:** If the economy starts with a higher (lower) physical-human capital ratio than the stationary solution, then the economy moves toward a balanced growth path with a lower (higher) level of human capital.

**c) The exogenous growth case:** Investment in human capital is insensitive to the physical-human capital ratio.

As shown in the appendix, the sign of the expression for the initial change in worked hours, 
\[ \Delta u = \left( u_2 / u_1 \right) \left( z^* - z^\tau \right) \]
provides sufficient information about the growth case displayed by our model around the balanced growth path. In such an expression, \((u_1, u_2, u_3, u_4)\) is the eigenvector associated with the negative eigenvalue in the dynamic system in \(z, u, \chi_1\) and \(\chi_2\), and \(z^*\) and \(z^\tau\) are the balanced-growth path values of the physical-human capital ratio associated with the untaxed and taxed economy, respectively. Following Ortigueira (1998), we should expect the economy to belong to the normal, exogenous or paradoxical growth case if \(\Delta u\) is either negative, zero or positive, respectively. For the numerical exercises discussed in section 3, we find that \(\Delta u < 0\) so the economies in discussion will always belong to the normal growth case.

### 3 A Quantitative Analysis of Tax Reforms

In this section we present numerical results for a series of alternative tax policies of the type \(\tau = \tau(\tau_k, \tau_l, \tau_c, \mu) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}^+\). For that purpose, we define a *feasible tax reform* as any change in two or more elements of the policy set \(\tau\) such that the corresponding present value of government revenue is equal to the present value of government spending. Throughout the numerical analysis, we assume that the government has access to a commitment technology that allows her to bind herself to the set of taxes originally announced.

Given the well-known results from the optimal taxation literature that aim for a zero capital income tax in the long-run (e.g. Chamley (1986), Lucas (1990), Chari
et al. (1991), and Jones et al. (1993), among others), our interest is to simulate economies where a tax on physical capital is replaced either by seigniorage or labor income taxation. Since the latter does not affect the properties of the economy along the balanced growth path, the long run effects of such a feasible reform on real variables are straightforward and given according to our previous discussion. Nevertheless, welfare effects along the balanced growth path of a tax reform involving changes in seigniorage are not so trivial: for example, a decrease in $r^*$ increases the long run physical-human capital ratio whereas an increase in $\mu$ has the opposite effect.

3.1 Calibration

We proceed to assign values for technology, preferences, and fiscal and monetary policy parameters. For the sake of clarity, we make the distinction between those parameters that are taken from the literature and those that are specially calibrated to match some observed variables for the U.S. economy. To facilitate the analysis, we follow Lucas (1990) and normalize initial output and the initial stock of human capital to unity.

Unless otherwise noticed, the following values for parameters are roughly standard in the literature (cf. Kydland and Prescott (1982)):

(i) $\alpha = 0.40$.

(ii) $\sigma = 1.5$.

(iii) $\rho = 0.04$.

(iv) $\psi = 0.83$. The parameter that determines the elasticity of substitution between cash and credit goods is fixed according to the value reported by Chari et al. (1991).

(v) $\mu = 0.068$. The number given to the rate of money growth roughly matches the growth rate for the U.S. monetary base in the post

---

7 Unfortunately, numeric exercises for this model show that a tax reform involving higher taxes on cash goods is not feasible in the sense that an increase in the consumption tax is not enough to compensate for the lost revenue derived from a decrease in the tax on capital income.

8 It may be shown that the optimal monetary policy in this model is given by the Friedman rule, namely that the after-tax nominal interest rate is zero. Hence, a natural concern is whether a tax reform involving changes in $\mu$ is such that the economy approaches the allocation implied by the Friedman rule. The optimal monetary policy $\mu_c$ in this model is given by $\mu_c = \delta + g - B$ (which is negative by the transversality condition). Hence, $\mu$ does not have an effect on the optimal monetary rule.

9 Along the paper, we define output in a narrowed way (i.e., excluding the production of human capital). For a method on how to estimate a broader measure of output, see Mulligan and Sala-i-Martin (1993).
World-War II period. Since the average rate of inflation for the U.S. is around 5.1 percent for the same period, equation (12) evaluated along the balanced growth path implies that the value assigned for $\mu$ above is roughly consistent with a 1.67 percent growth rate in per capita gross national product (see below).

(vi) $\tau_i = 0.43, \tau_l = 0.25$ and $\tau_c = 0.06$. These fiscal parameter values are just the averages for the effective tax rates in the U.S. on physical capital income, labor income and consumption respectively, as reported in Mendoza et al. (1994) for the period 1965-1988.10

(vii) $G/h = 0.21$. Given our normalization for initial output and human capital, this value means that 21 percent of output is devoted to government consumption of goods and services along the balanced growth path, as in Lucas (1990).

Parameters that are calibrated to match average values for the U.S. economy are the following:

(i) $A = 1.299$. The technology parameter $A$ is conveniently fixed so that initial output along the balanced growth path is set equal to 1. The corresponding physical capital-output ratio is 2.2, which is slightly smaller than the value obtained by Kydland and Prescott (1982).

(ii) $\delta = 0.06$. The parameter for physical and human capital depreciation is set so that we may get a physical investment-output ratio of 0.166, which roughly matches the average value for fixed nonresidential investment over output observed over the postwar period, as reported in Cooley and Hansen (1992). This value for human capital depreciation is also well within the interval suggested by Stokey and Rebelo (1995).

(iii) $B = 0.125$. Productivity of human capital $B$ is set so that the long-run growth rate of per capita income equals 1.67 percent (cf Lucas (1990)). This value for $B$ is also reported in Ortigueira (1998).

(iv) $\eta = 0.46$. As a reference, Cooley and Hansen (1992) report values of $\eta = 0.83$ and $\eta = 0.50$ for a logarithmic utility function with indivisible labor. Alternatively, Chari et al. (1991) estimate a value of $\eta = 0.43$. Thus we choose to set the share of the cash good in total consumption equal to 0.46, which is just the intermediate value.

---

10 As a reference, Cooley and Hansen (1992) computations set $\tau_i = 0.50, \tau_l = 0.23$ and $\tau_c = 0$. 

12
between $\eta = 0.43$ and $\eta = 0.5$.\footnote{We decided not to use a consumption-velocity equation to calibrate the values for $\eta$ and $\psi$ since our definition of "money" is the monetary base. As is well known (cf. Hodrick et al. (1991)), velocity appears to be non-stationary when either M1 or the monetary base is used as the relevant monetary aggregate.} This estimate for $\eta$ yields a ratio of seigniorage revenue to output equal to 0.50 in percentage terms, which compares favorably with the values over the 0.49–0.83 percent interval reported in Cooley and Hansen (1991) for a 5 percent inflation rate.

Finally, transfer payments $T/h$ are residually defined so that revenue equals government spending along the balanced growth path, according to (10).\footnote{Although it is not possible to know the value of $T/h$ along the balanced growth path without first calculating the dynamics of transition, in a first attempt we follow Lucas (1990) by assuming that debt is neither accumulated nor decumulated along the transition. Of course, we will relax this assumption once we estimate welfare along the transition path.} For the benchmark economy, total revenue is equal to 28.9 percent of total output, so $T/h = 0.079$. This number is closely similar to the 0.0726 value reported in Jones et al. (1993) when transfer payments are adjusted for social security payments.

To summarize, table 1 presents the parameter values for our benchmark economy.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Parameter Values for the Benchmark Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Technology parameters</strong></td>
<td>$\alpha = 0.40, \delta = 0.06, A = 1.299, B = 0.125$</td>
</tr>
<tr>
<td><strong>Preference parameters</strong></td>
<td>$\rho = 0.04, \sigma = 1.5, \eta = 0.46, \psi = 0.83$</td>
</tr>
<tr>
<td><strong>Monetary and fiscal policy parameters</strong></td>
<td>$\mu = 0.068, \tau_c = 0.43, \tau_l = 0.25, \tau_e = 0.06, G/h = 0.21, T/h = 0.079$</td>
</tr>
</tbody>
</table>

3.2 Estimation

In this part we present a quantitative analysis on the welfare cost of alternative tax policies that are designed so that the government budget constraint holds. As mentioned earlier, we are interested in policy reforms where a decrease in the tax on physical capital income is replaced by increases either in seigniorage or labor income taxes. Since we have six parameters related to monetary and fiscal policies, numeric exercises for this part involve treating one of the variables as
endogenous (specifically, either the rate of money growth $\mu$ or the labor income tax $\tau_0$), given the values of the other five parameters.

As in Cooley and Hansen (1992), we provide welfare estimates both along the balanced growth path as well as along the transition. This last exercise is not innocuous: As illustrated by Lucas (1990) and Ortigueira (1998), neglecting transitional dynamics effects in welfare analysis may lead to substantial overestimation in the results. It turns out that the numeric difference in results is quite substantial in our model.

To figure out the effects of a tax policy, we imagine an untaxed economy that is along its balanced growth path. We denote $(c_1^*(t), c_2^*(t))$ as the corresponding consumption path, and $k^*(0)$ and $h^*(0)$ as the initial endowments of physical and human capital. As before, $C^*(t)$ denotes the associated aggregate consumption with no taxes. At time zero, an unexpected tax policy $\tau = \tau(k_0, \tau_1, \tau_c, \mu)$ is introduced so that the economy moves out of its initial allocation in order to converge to its new balanced growth path (provided it exists). The corresponding consumption path for the taxed economy is denoted as $(c_1^*(t), c_2^*(t))$. Accordingly, aggregate consumption (off the balanced growth path) under tax policy $\tau$ is expressed as $C_\tau(t)$. We assume that the household has perfect foresight once the tax policy is announced.

Following Lucas (1987) and Cooley and Hansen (1992), we measure the welfare cost of policy $\tau = \tau(k_0, \tau_1, \tau_c, \mu)$ as the percentage increase in aggregate consumption required so that the household is indifferent between the taxed allocation $(c_1^*(t), c_2^*(t))$ under policy $\tau$ and the lump-sum tax allocation $(c_1^*(t), c_2^*(t))$ associated with a zero tax policy of the type $\tau_0 = \tau_0(k_0 = \tau_1 = \tau_c = \mu = 0)$.\(^\dagger\) Hence the welfare cost of tax policy $\tau$ is the compensation payment $\gamma$ (in aggregate consumption terms) that satisfies:

$$
\int_0^\infty e^{-\rho t} \frac{(1+\gamma)C_\tau(t)^{1-\sigma} - 1}{1-\sigma} dt = \int_0^\infty e^{-\rho t} \frac{(c_\tau^*(t))^{1-\sigma} - 1}{1-\sigma} dt
$$

Following Ortigueira (1998), we may interpret the consumption path $(c_1^*(t), c_2^*(t))$ starting at $k^*(0)$ and $h^*(0)$ as the path originated in the stationary taxed economy which experiences a perturbation on physical and human capital that places the economy at the initial conditions $k^*(0)$ and $h^*(0)$.

\(^\dagger\) Notice that tax policy $\tau_0$ is not Pareto optimal.
Of course, the balanced growth path version of equation (16) is given by:

$$U((1 + \gamma)C_\tau^*(t)) = U(C^*(t))$$

(17)

where $C_\tau^*(t)$ is the aggregate consumption under tax policy $\tau$ along the balanced growth path.

To make our results readily comparable with those of Cooley and Hansen (1992), we also calculate the welfare cost of tax policy $\gamma$ in terms of narrow output. In particular, from the value of $\gamma$ that satisfies either (16) or (17), we estimate $\gamma(c_1(t) + c_2(t))$. The welfare cost in terms of income is thus the value of such expression denoted as a percentage of narrow output defined under policy $\tau$.

In addition, we need to check if the tax reform announced is feasible. The steps involved in the computation are as follows: First, we start with a policy guess $\tau^*$ (for given $\tau$ and changes in either $\tau_1$ or $\mu$) and calculate the present value of the corresponding government revenue along the transition path under taxes. Next, since the policy guess $\tau^*$ usually defines a new lump-sum transfer payment $T/\mu$ along the balanced growth path according to (10), we compute the corresponding present value for total government spending also along the taxed transition path. Finally, we check whether the present value of government revenue is equal to the present value of government spending. If it is not (as it might be expected), we continue adjusting the free tax policy parameter (either $\mu$ or $\tau$) until the tax reform is feasible.

As discussed earlier, the short run effects of a tax reform on real variables may shed some additional insight about the transition. As shown in the appendix, these short run effects on working hours, cash and credit goods once the new tax policy is announced are given by:

$$\frac{u_\tau(0) - u^*(0)}{u^*(0)} = \frac{\Delta u}{u^*(0)},$$

(18)

$$\frac{c_1(0) - c_1^*(0)}{c_1^*(0)} = \frac{(\chi_1^* - \chi_1^*(0)) + \Delta \chi_1}{\chi_1(0)}$$

(19)

and

$$\frac{c_2(0) - c_2^*(0)}{c_2^*(0)} = \frac{(\chi_2^* - \chi_2^*(0)) + \Delta \chi_2}{\chi_2(0)}$$

(20)
Before presenting the results, it is worthwhile to make two remarks. First, for all the numerical exercises considered in this article there is a unique negative eigenvalue. In other words, the dynamic system is (locally) saddle-path stable with a unique stable manifold. Second, we find that the sign of $\Delta u$ in expression (18) is always negative for all the numerical exercises involved. Therefore, we may be confident that the economy belongs to the normal growth case for all the exercises shown below. In other words, the economy always converges to a balanced growth path with a higher level of physical and human capital.

### 3.3 Results

Now we proceed to estimate the welfare cost of a tax reform along the balanced growth path first, according to (17). Estimates including transitional dynamic effects are provided latter. The first exercise involves changes in the tax on physical capital income that are compensated by changes in the labor income tax. It is important to recall that, given the assumptions of the model, a tax on labor income basically acts like a lump-sum tax along the balanced growth path. For comparison purposes, welfare costs are presented either in terms of consumption or output, denoted in percentage terms as $\gamma_c$ or $\gamma_y$, respectively. The first row of table 2 depicts a welfare cost of taxation of 24.7 percent in consumption terms and of 15.4 percent in output terms under the benchmark economy. The first two numbers of the second row simply state that a decrease in the physical capital income tax from 0.43 to 0.40 needs to be compensated by an increase in labor taxation from 0.25 to 0.255 so that government revenue remains constant. Since labor taxation has no effect on real variables along the balanced growth path, the cost of taxation monotonically decreases as $r^*$ decreases. The welfare cost at $r^* = 0$ is strictly positive since there are seigniorage and consumption taxes still in place. However, they are trivially small if compared to the initial welfare cost.

The second part of table 2 shows exercises where alternative decreases in $r_k$ are compensated by higher seigniorage. To facilitate the analysis, the welfare cost under the benchmark is shown again in the first row. There are two basic observations in this case. First and not surprisingly, welfare cost estimates are now greater given the distorting nature of $\mu$. More important, the welfare cost (either in consumption or income terms) monotonically decreases as $r_k$ decreases. In other words, substitution of capital income taxation for seigniorage revenue is unambiguously (and significantly) welfare improving. In principle, this welfare result is non-trivial given the opposite effects of such policy on real variables along the balanced growth path as mentioned earlier.
Table 2
Welfare Cost of Alternative Feasible Tax Reforms along the Balanced Growth Path
Benchmark Economy

A. Labor Taxation

<table>
<thead>
<tr>
<th>$\tau_k$</th>
<th>$\tau_l$</th>
<th>$\gamma_c (%)$</th>
<th>$\gamma_f (%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Base policy</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.43</td>
<td>0.25</td>
<td>24.7</td>
<td>15.4</td>
</tr>
<tr>
<td><strong>Replace $\tau_l$ by $\tau_l$</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>0.255</td>
<td>21.6</td>
<td>13.4</td>
</tr>
<tr>
<td>0.30</td>
<td>0.274</td>
<td>13.5</td>
<td>8.3</td>
</tr>
<tr>
<td>0.20</td>
<td>0.295</td>
<td>8.0</td>
<td>4.9</td>
</tr>
<tr>
<td>0.10</td>
<td>0.317</td>
<td>4.0</td>
<td>2.4</td>
</tr>
<tr>
<td>0.0</td>
<td>0.339</td>
<td>1.1</td>
<td>0.6</td>
</tr>
</tbody>
</table>

B. Taxation on Money Holdings

<table>
<thead>
<tr>
<th>$\tau_k$</th>
<th>$\mu$</th>
<th>$\gamma_c (%)$</th>
<th>$\gamma_f (%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Base policy</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.43</td>
<td>0.068</td>
<td>24.7</td>
<td>15.4</td>
</tr>
<tr>
<td><strong>Replace $\tau_k$ by $\mu$</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.40</td>
<td>0.088</td>
<td>22.5</td>
<td>14.0</td>
</tr>
<tr>
<td>0.30</td>
<td>0.186</td>
<td>17.1</td>
<td>10.6</td>
</tr>
<tr>
<td>0.20</td>
<td>0.338</td>
<td>13.2</td>
<td>8.1</td>
</tr>
<tr>
<td>0.10</td>
<td>0.537</td>
<td>9.5</td>
<td>5.8</td>
</tr>
<tr>
<td>0.0</td>
<td>0.771</td>
<td>5.7</td>
<td>3.4</td>
</tr>
</tbody>
</table>

At this point, it is worthwhile to compare our results with those obtained by Cooley and Hansen (1992). For their benchmark policy \( \{\tau_k = 0.5, \tau_l = 0.23, \tau_c = 0, \mu = 0\} \), they report a welfare cost of 13.3 percent in output terms, which may be decreased to either 7.8 or 6.7 percent as the tax on physical capital income is eliminated and replaced either by labor taxation or seigniorage, respectively. In contrast to our results, they find that a tax on money holdings is better than a tax on labor income in welfare terms. In their model, this result is not trivial since they include indivisible labor in the utility function, so taxes on labor income are in fact distorting. The authors also find a lower welfare gain (in percentage points) of moving from the benchmark policy to a zero tax policy on

---

14 Interestingly, if we set $\mu = 0$ in our model, we get a welfare cost of 13.2 percent in terms of output for the benchmark economy.
physical capital. For example, the welfare gain of eliminating the tax on physical capital income under seigniorage in our model is almost twice as that found in Cooley and Hansen.

Now we estimate welfare costs of feasible tax reforms incorporating transitional effects. As is well known (cf. Lucas (1990)), moving towards a balanced growth path as a result of decreases in $\tau_k$ involves a transitional period that affects the initial allocation of consumption and working hours downward. Therefore, this effect should at least partially offset the welfare gain enjoyed along the new balanced growth path. In other words, without any further analysis, the welfare gain results shown in table 2 are presumably overestimated. In addition, changes in either $\mu$ or $\tau_k$ may affect the transition towards the new balanced growth path (and thus affect welfare) through its effects on the law of motion for cash and credit goods and the supply of working time as well, as discussed earlier.

Table 3 shows the welfare costs of feasible tax reforms (in consumption and income terms) when transitional dynamics are taken into account.\textsuperscript{15} We also display the short-run effects of taxation on working hours and the consumption of cash and credit goods as given by (18) – (20). Part A shows the effect of an increase in labor taxation when physical capital taxation is decreased. There are several interesting things to notice. First, compared to table 2, a feasible tax reform under transition requires now a higher tax on labor for every level of $\tau_k$. More important, gradual decreases in capital income taxation yield a monotonically lower welfare cost. As expected, at $\tau_k = 0$ the welfare cost in the last row of part A is the same as the one reported in table 2 (since there is no perturbation to capital income and the labor income tax is non-distorting when the economy is along its balanced growth path). The last three columns finally show that the short run effects on the consumption of cash and credit goods as well as worked hours are monotonic. Overall, it is evident that the welfare gain of moving from $\tau_k = 0.43$ to a zero capital income tax is now substantially lower once the transition is taken into account.\textsuperscript{16}

Part B of table 3 shows a similar exercise when lost revenue must be replaced by increases in seigniorage. Here, transitional dynamics results are more interesting. Contrary to what is found in the balanced growth path analysis, the welfare cost of taxation increases relative to its benchmark value (either in

\textsuperscript{15} Given the approximation methods around the taxed balanced growth path, it is possible that approximation errors for the welfare cost of taxation for high values of $\tau_k$ are sizable.
\textsuperscript{16} Interestingly, Cooley and Hansen (1992) also find a welfare gain along the transition of almost 2.1 percentage points in output terms when replacing a tax on capital income (from 50 to zero percent) for a labor income tax.
Table 3
Welfare Cost of Alternative Feasible Tax Reforms along the Transition Path
Benchmark Economy

A. Labor Taxation

<table>
<thead>
<tr>
<th>$\tau_k$</th>
<th>$\tau_l$</th>
<th>$\gamma_c$ (%)</th>
<th>$\gamma_y$ (%)</th>
<th>$\frac{c_{2t}(0) - c'_1(0)}{c_1(0)}$</th>
<th>$\frac{c_{2r}(0) - c'_2(0)}{c'_2(0)}$</th>
<th>$\frac{u_r(0) - u^<em>(0)}{u^</em>(0)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base policy</td>
<td>0.43</td>
<td>0.25</td>
<td>4.2</td>
<td>2.7</td>
<td>-0.322</td>
<td>0.313</td>
</tr>
<tr>
<td>Replace $\tau_k$ by $\tau_l$</td>
<td>0.40</td>
<td>0.256</td>
<td>3.9</td>
<td>2.5</td>
<td>-0.335</td>
<td>0.294</td>
</tr>
<tr>
<td></td>
<td>0.30</td>
<td>0.279</td>
<td>2.9</td>
<td>1.8</td>
<td>-0.369</td>
<td>0.242</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>0.307</td>
<td>2.1</td>
<td>1.3</td>
<td>-0.398</td>
<td>0.200</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>0.338</td>
<td>1.5</td>
<td>0.9</td>
<td>-0.423</td>
<td>0.164</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>0.372</td>
<td>1.1</td>
<td>0.6</td>
<td>-0.445</td>
<td>0.131</td>
</tr>
</tbody>
</table>

B. Taxation on Money Holdings

<table>
<thead>
<tr>
<th>$\tau_k$</th>
<th>$\mu$</th>
<th>$\gamma_c$ (%)</th>
<th>$\gamma_y$ (%)</th>
<th>$\frac{c_{2t}(0) - c'_1(0)}{c_1(0)}$</th>
<th>$\frac{c_{2r}(0) - c'_2(0)}{c'_2(0)}$</th>
<th>$\frac{u_r(0) - u^<em>(0)}{u^</em>(0)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base policy</td>
<td>0.43</td>
<td>0.068</td>
<td>4.2</td>
<td>2.7</td>
<td>-0.322</td>
<td>0.313</td>
</tr>
<tr>
<td>Replace $\tau_k$ by $\mu$</td>
<td>0.40</td>
<td>0.093</td>
<td>4.3</td>
<td>2.8</td>
<td>-0.404</td>
<td>0.321</td>
</tr>
<tr>
<td></td>
<td>0.30</td>
<td>0.214</td>
<td>4.9</td>
<td>3.1</td>
<td>-0.667</td>
<td>0.352</td>
</tr>
<tr>
<td></td>
<td>0.20</td>
<td>0.406</td>
<td>5.7</td>
<td>3.6</td>
<td>-0.855</td>
<td>0.364</td>
</tr>
<tr>
<td></td>
<td>0.10</td>
<td>0.677</td>
<td>6.1</td>
<td>3.8</td>
<td>-0.948</td>
<td>0.342</td>
</tr>
<tr>
<td></td>
<td>0.00</td>
<td>1.064</td>
<td>6.2</td>
<td>3.7</td>
<td>-0.985</td>
<td>0.290</td>
</tr>
</tbody>
</table>

Consumption (or income terms) for all the simulations reported. In other words, we find that starting from the initial policy and moving towards zero capital income taxation is not welfare improving for the simulations under study.\(^{17}\) In fact, part B of

\(^{17}\) The fact that higher values for $\mu$ are needed under transitional dynamics than under the balanced-growth path analysis to compensate for decreases in $\tau_k$ does not explain by itself the increases in the welfare cost of taxation obtained here. For example, transitional dynamic simulations (not shown) with values for $\mu$ obtained under the balanced growth path in table 2 still show an increase in the welfare cost for every $\tau_k$. 
Table 3 shows a welfare loss of 1 percentage point in GDP terms (a 2 percent loss if measured in consumption terms) of such a tax reform.

According to the results reported in table 3, labor income taxation is far better than seigniorage in terms of welfare costs. As for the balanced growth path case, this result is also different from what is found in Cooley and Hansen (1992). As noted before, this is partially due to the fact that labor income taxation has no effect on balanced-growth path variables, but also to its (presumably) relative small effect along the transition. However, a second major difference in the transitional dynamic results with respect to Cooley and Hansen (1992) is that moving towards a zero physical capital income tax and substituting lost revenue by increases in $\mu$ is welfare improving in their model, the welfare gain being around 2.7 percentage points in output terms.

The natural step now is to check whether the increasing welfare cost found in the second part of table 3 may also be reproduced under alternative parameter values, even along the balanced growth path. Table 4 provides an answer to both concerns. Welfare costs (in both consumption and income terms) along the balanced growth path as well as along the transition are provided with alternative parameter values for $\eta$, $\sigma$ and $\psi$ under the benchmark economy. For example, the first set of exercises involves a feasible tax reform with alternative values for $\eta$ and $\psi$ only. The second and fifth column simply shows the corresponding values for $\mu$ consistent with a feasible tax reform under the balanced growth and transition path, respectively.

There are three important observations derived from table 4. First, welfare costs along the balanced growth path are always decreasing as $\eta$ goes to zero, regardless of how they are measured. Second, the former result of increasing welfare costs along the transition (as $\psi$ decreases) cannot be generalized in our model. For example, this increasing pattern is consistent with alternative values of $\sigma$, but opposite or mixed results are obtained when either $\eta$ or $\psi$ are changed, respectively. Finally, welfare estimates are particularly sensitive to the share $\eta$ of cash goods in total consumption: welfare costs are nearly zero when capital income is not taxed either under $\eta = 0.25$ or $\eta = 0.83$.

As mentioned earlier, under an exogenous growth framework Cooley and Hansen (1992) perform a feasible tax reform for $\tau_k = 0$ and higher taxes on money holdings to find that welfare decreases with respect to the benchmark case. So, according to our results, it is an open question whether their results would still be valid under alternative parameter values. Alternatively, we could check if our results depend upon our endogenous growth specification or the assumption that taxes must be paid with money.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Balanced growth path</th>
<th>Transitional dynamics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu$</td>
<td>$\gamma_c$ (%)</td>
</tr>
<tr>
<td>$\eta=0.25$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_k=0.43$</td>
<td>0.068</td>
<td>23.4</td>
</tr>
<tr>
<td>$r_k=0.20$</td>
<td>0.326</td>
<td>9.1</td>
</tr>
<tr>
<td>$r_k=0.0$</td>
<td>0.678</td>
<td>0.0</td>
</tr>
<tr>
<td>$\eta=0.83$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_k=0.43$</td>
<td>0.068</td>
<td>23.4</td>
</tr>
<tr>
<td>$r_k=0.20$</td>
<td>0.096</td>
<td>7.0</td>
</tr>
<tr>
<td>$r_k=0.0$</td>
<td>0.134</td>
<td>0.0</td>
</tr>
<tr>
<td>$\sigma=1.01$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_k=0.43$</td>
<td>0.068</td>
<td>22.3</td>
</tr>
<tr>
<td>$r_k=0.20$</td>
<td>0.343</td>
<td>12.2</td>
</tr>
<tr>
<td>$r_k=0.0$</td>
<td>0.779</td>
<td>5.7</td>
</tr>
<tr>
<td>$\sigma=3.0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_k=0.43$</td>
<td>0.068</td>
<td>27.1</td>
</tr>
<tr>
<td>$r_k=0.20$</td>
<td>0.333</td>
<td>14.2</td>
</tr>
<tr>
<td>$r_k=0.0$</td>
<td>0.764</td>
<td>5.7</td>
</tr>
<tr>
<td>$\sigma=5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_k=0.43$</td>
<td>0.068</td>
<td>28.0</td>
</tr>
<tr>
<td>$r_k=0.20$</td>
<td>0.330</td>
<td>14.6</td>
</tr>
<tr>
<td>$r_k=0.0$</td>
<td>0.759</td>
<td>5.6</td>
</tr>
<tr>
<td>$\psi=0.5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_k=0.43$</td>
<td>0.068</td>
<td>24.1</td>
</tr>
<tr>
<td>$r_k=0.20$</td>
<td>0.170</td>
<td>9.2</td>
</tr>
<tr>
<td>$r_k=0.0$</td>
<td>0.323</td>
<td>3.1</td>
</tr>
<tr>
<td>$\psi=0.01$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_k=0.43$</td>
<td>0.068</td>
<td>23.8</td>
</tr>
<tr>
<td>$r_k=0.20$</td>
<td>0.146</td>
<td>8.2</td>
</tr>
<tr>
<td>$r_k=0.0$</td>
<td>0.246</td>
<td>1.2</td>
</tr>
<tr>
<td>$\psi=-0.5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r_k=0.43$</td>
<td>0.068</td>
<td>23.6</td>
</tr>
<tr>
<td>$r_k=0.20$</td>
<td>0.139</td>
<td>4.8</td>
</tr>
<tr>
<td>$r_k=0.0$</td>
<td>0.228</td>
<td>0.7</td>
</tr>
</tbody>
</table>
Unfortunately, in our model it is hard to say analytically under which conditions a feasible tax reform is welfare improving. Nonetheless, from table 5 we may infer that none of these assumptions—either the endogenous growth or taxes-paid-with-money specifications—is responsible for explaining by itself the increasing welfare cost of a tax reform. Part A calculates the welfare cost along the transition assuming zero exogenous growth while keeping the assumption that taxes must be paid with money. The second exercise (depicted in part B) is based on the opposite scenario, namely money is not needed to pay for taxes but growth is endogenous. As table 5 illustrates, we still get an increasing welfare cost in each case as \( \tau \) goes to zero.

By looking at the results of tables 3 and 5, it is useful to compare the welfare implications of our model assumptions. As expected, if taxes need to be paid with money the economy should borne higher welfare costs at each level of taxation. In such case, welfare costs are 12 percent higher in consumption terms on average, an increase which is far from being disregarded. On the other hand, the endogenous
growth model considered here should bring about higher welfare costs than standard exogenous growth models, as discussed in the introduction. Assuming that both economies are constrained to pay taxes with money, we find that the intuition is proved to be right with a substantial difference in welfare estimates (a difference of about 60 percent in consumption terms on average).

Finally, to prove the sensitivity of our welfare estimates to the value given to $\eta$, we also calculate welfare costs (not shown) of feasible tax reforms under an exogenous growth model without the assumption that taxes must be paid with money (i.e., the Cooley and Hansen’s case). It turns out that, under this specification, welfare is decreasing for $\eta = 0.83$ (the same parameter value as in Cooley and Hansen) but again increasing if $\eta = 0.46$. Hence, we may safely argue that the increasing welfare cost found in this paper depends heavily on the values given to the share of cash goods in total consumption.

4. Conclusion

This paper is concerned about the welfare effects of alternative feasible tax reforms involving both fiscal and monetary policies in an endogenous growth context. We find sizable welfare gains from eliminating capital income taxation along the balanced growth path regardless of how lost revenue is raised. Once the initial impact on consumption and worked hours are taken into account, the welfare gain of zero capital income taxation is significantly reduced, although still positive when revenue is raised through labor income taxation. But if revenue is raised by increases in the growth rate of money, such welfare gain now becomes negative, contrary to what is found in Cooley and Hansen (1992). As discussed in the text, the share of cash goods in total consumption is crucial in obtaining this result. On the other hand, we also find that our extra assumptions—the endogenous growth framework and the need to pay taxes with money—bring substantial differences in welfare if compared to a standard exogenous growth model.

As this paper has also shown, ignoring transitional dynamic effects may lead to the wrong conclusion—both in qualitative and quantitative terms—about the welfare consequences of a tax reform. This result thus reinforces the points made earlier by Lucas (1990) and Ortigueira (1998) about the importance of considering the transition in welfare analysis. In our model, this is particularly evident when a tax reform involves higher taxes on money holdings. In this sense, it is important to emphasize that increases in welfare cost under such circumstances are perfectly possible: a feasible tax reform with higher distorting taxes and no taxes on capital income does not necessarily lead the economy towards a Pareto optimal allocation. For example, such an exercise would need to be accompanied by negative taxes on money holdings.
Overall, the analysis thus far has shown that labor taxation is far better than seigniorage revenue in terms of welfare for all the tax reform exercises considered. This is in part a natural result of the Uzawa-Lucas model, where labor taxation is in fact a lump-sum tax in the long run. Natural extensions of this analysis would include a more general model like the one proposed by Rebelo (1991), where physical capital is needed for the production of human capital, or the explicit introduction of leisure in the utility function, as in Jones and Manuelli (1995). Since labor income taxation may be distorting along the balanced growth path under such situations, now we should presumably expect a lower welfare gain from a decrease in the tax on physical capital income. On the other hand, these extra assumptions allow for taxation policies to have an impact on the endogenous growth rate of the economy. Under all these conditions, it would be interesting to check the robustness of our results.

Appendix

The purpose of this section is to get expressions for the short-run effect of a tax reform on the consumption of cash goods, credit goods and worked hours. Let us first denote the balanced-growth path values of the untaxed economy by \( z^* \), \( x^*_1 \) and \( x^*_2 \), and the corresponding balanced-growth path values for a particular tax policy \( \tau \) by \( z^*_\tau \), \( x^*_1\tau \) and \( x^*_2\tau \). Imagine that at time zero a feasible tax reform is introduced. The corresponding initial endowments of physical and human capital are denoted by \( k^*(0) \) and \( h^*(0) \), respectively. The untaxed path for cash and credit goods may be simply re-expressed as:

\[
\begin{align*}
 c^*_1(t) &= \frac{c^*_1(t)}{h^*_1(t)} \quad h^*_1(t) = x^*_1h^*(0)e^{g_t} \\
 c^*_2(t) &= \frac{c^*_2(t)}{h^*_2(t)} \quad h^*_2(t) = x^*_2h^*(0)e^{g_t}
\end{align*}
\]  

(A1)  

(A2)

Following Ortigueira (1998), it is possible to show that the consumption path for cash and credit goods after the introduction of taxes in a neighborhood of the balanced-growth path are respectively given by:

\[
\begin{align*}
 x^*_1\tau(t) &= (x^*_1 + e^{\Delta t} A x^*_1)h^*(0)e^{g_t} e^{-B_t e^{a_t - 1} / \beta}
\end{align*}
\]  

(A3)

and
\[ X_{2r}(t) = (X_{2r} + e^{\beta t} \Delta X_2) h^*(0) e^{zt} e^{-\beta \Delta u(t)} / \beta \] (A4)

where \( \Delta X_1 = (\nu_2 / \nu_1)(z^* - z^*_1) \), \( \Delta X_2 = (\nu_4 / \nu_1)(z^* - z^*_2) \), \( \Delta u = (\nu_2 / \nu_1)(z^* - z^*_1) \), and \( (\nu_1, \nu_2, \nu_3, \nu_4) \) is the eigenvector associated with the (unique) negative eigenvalue \( \beta \) in the dynamic system on \( z_n, u_n, X_1r \) and \( X_2r \).

Given (A1) – (A4) and using the fact that \( u_r(t) = u^* + e^{\beta t} \Delta u \), we may express the initial shock of a feasible tax reform on \( u, X_1 \) and \( X_2 \) as:

\[ \frac{u_r(0) - u^*(0)}{u^*(0)} = \frac{\Delta u}{u^*(0)}, \] (A5)

\[ \frac{c^*_1(0) - c^*_1(0)}{c^*_1(0)} = \frac{(X^*_1 - X^*_1(0)) + \Delta X_1}{X^*_1(0)}, \] (A6)

and

\[ \frac{c^*_2(0) - c^*_2(0)}{c^*_2(0)} = \frac{(X^*_2 - X^*_2(0)) + \Delta X_2}{X^*_2(0)} \] (A7)

Equations (A5)-(A7) appear as equations (18)-(20) in the text. Each of the expressions (A6) and (A7) has two components. The term in parenthesis captures the long-run effect of the feasible tax reform, whereas the delta term shows the short-run impact of such policy. Note that the first term disappears in (A5) since working hours are not affected neither by fiscal nor monetary policy variables along the balanced growth path.
References


Documentos de trabajo de reciente aparición

División de Administración Pública

Guerrero Amparan Juan Pablo y Helena Hofbauer Balmori. Índice de transparencia presupuestaria en cinco países de América Latina. AP-113

Gryj Rubenstein, Lina y Juan Pablo, Guerrero Amparan. Las reformas municipales en el estado de Guanajuato. AP-114

Guerrero Amparan, Juan Pablo y Sánchez de la Vega Rodolfo Madrid. Consideraciones sobre la transparencia en el gasto público en México. AP-115

Gryj Rubenstein, Lina y Juan Pablo, Guerrero Amparan. La reforma municipal en el estado de Zacatecas. AP-116

Arellano Gault, David. La transformación de la administración pública en México: Limites y posibilidades de un servicio civil de carrera. AP-117

Carter, Nicole y Leonard, Ortolano. Subsidies for Public Services at an International Border: Implementing Government Assistance for Environmental Infrastructure in Texas Colonias. AP-118


Del Castillo, Arturo. BUREAUCRACY and CORRUPTION. An Organizational Perspective. AP-120

Arellano, David, Coronilla, Efrain, Coronilla, Raúl y Alberto Santibáñez. Hacia una política de transporte en el Distrito Federal: propuestas de reforma institucional y organizacional. AP-121

División de Economía

Hernández, Fausto, Pagán, José Luis y Julia Paxton. Start up Capital, Microenterprises and Technical Efficiency in Mexico. E-226

Ramírez, José Carlos y Rogelio, Sandoval. Patrones no lineales en los rendimientos de las acciones de la BMV: una prueba basada en cadenas de Markov de segundo orden. E-227

Brito, Dagobert L. y Juan, Rosellón. A General Equilibrium Model of Pricing Natural Gas in Mexico. E-228

Ramírez, José y Juan, Rosellón. Pricing Natural Gas Distribution in Mexico. E-229

Brito, Dagobert L y Juan, Rosellón. A Solar Power Project in Mexico for the California Electricity Market. E-230

Cordourier, Gabriela y Gómez-Galvarriato, Aurora. La evolución de la participación laboral de las mujeres en la industria en México: una visión de largo plazo. E-231


Del Ángel, Gustavo. Historiografía reciente de la banca en México. Siglos XIX y XX. E-233

Unger, Kurt. Determinantes de las exportaciones manufactureras mexicanas y su sensibilidad a la productividad, el tipo de cambio e importaciones relacionadas. Evidencias preliminares. E-234
División de Estudios Internacionales

Schiavon, Jorge A., *Sobre contagios y remedios: la heterodoxia económica del New Deal, la política exterior corrección de Roosevelt y su impacto sobre la administración cardenista*. EI-81


Jones, Adam, *Genocide and Humanitarian Intervention: Incorporating the Gender Variable*. EI-83


Velasco, Jesús, *Caminando por la historia intelectual de Seymour Martin Lipset*. EI-86

Chabat, Jorge, *The Combat of Drug Trafficking in Mexico under Salinas: The Limits of Tolerance*. EI-87

Chabat, Jorge, *Mexico's War on Drugs: No Margin for Maneuver*. EI-88

Schiavon, Jorge A., *International Relations and Comparative Politics: Cooperation or Conflict?*. EI-89

Jones, Adam, *Reforming the International Financial Institutions*. EI-90


División de Estudios Políticos


Bataillon, Gilles, *Guerra y Asamblea*. EP-144


División de Historia


Meyer, Jean, *¿Quiénes son esos hombres?*. H-11.


Favre, Henri, *Chiapas 1993: intento de análisis de una situación de insurrección*. H-14

Pipitone, Ugo, *La región europea en formación*. H-15

Meyer, Jean, *Guerra, violencia y religión*. H-16

Meyer, Jean, *Guerra, religión y violencia, el contexto salvadoreño de la muerte de Monseñor Romero*. H-17

Pipitone, Ugo, *Caos y Globalización*. H-18

